FURTHER RELATIONS OF MAGNETIC, GRAVITATING, AND LUMINOUS FORCE.

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(Read before the American Philosophical Society, June 18, 1875.)

Ohm's law is a particular case of the general principle that an impulsive force may be measured by the product of resistance overcome, by velocity communicated. Illustrations of this principle may be found

In General Physics,	Mass	$\times$ Velocity	$=$ Momentum ;
In Electricity,	Resistance	$\times$ Current	$=$ Electromotive force ;
In Chemistry,	Atomic weight	$\times$ Specific heat	$=$ Chemical constant ;
In Cosmogony,	Inertia-resistance	$\times$ Gravitating impulse	$=$ Centripetal constant

Since all these expressions refer to actions from or towards given centres, they are governed by the same mathematical relations, and the different names by which the activities are designated, do not necessarily imply any differences in the nature of the forces themselves.

Clerk Maxwell asks,* "Is it possible that the attraction of [the Sun and Moon], by causing strains in the interior of the earth, produces changes in the magnetism of the Earth, and so by a kind of tidal action causes the semidiurnal variations?" Eleven years ago, in the paper which received the Magellanic gold medal, and in other communications,† I anticipated the question, and gave reasons for answering it in the affirmative.

If the primary disturbance is of a tidal character, it does not involve the entire force of $[M]$, but merely the differential force $[ML^{-3}]$. If we regard the electric $[M^{\frac{1}{2}}]$ as really representing $[M^{\frac{1}{2}}L^{-\frac{3}{2}}]$, each of the mass-factors in Maxwell's table of electrostatic and electromagnetic dimensions should be multiplied by $[L^{-\frac{3}{2}}]$. This multiplication produces a *precise correspondence* between the electrical and gravitating fields, both in extent, and in many suggestive details.

In my communication on the "Velocity of Primitive Undulations,"‡ I showed that the present numerical value of the velocity-ratio, lt^{-1}, at its upper limit, or the limit between total solar dissociation and incipient aggregation, is the velocity of light, and that the planetary ratios are also in close accordance with the ratio between the radius of gyration of the solar system when nebulously diffused, and Sun's radius of gyration about the centre of gravity of the system. If we wish to extend our comparisons to the lower limit, or the limit between total aggregation and commencing dissociation, the directions of v_{ι} and $v_{\iota\iota}$ should be taken tangentially instead of radially. Designating the symbols for the lower limit by enclosures we have $(v_{\iota}) = v_{\iota} \div \sqrt{2}$; $(v_{\iota\iota}) = \dfrac{\pi}{2} v_{\iota\iota}$; $(lt^{-1}) = (v_{\iota} \div v_{\iota\iota}) = \dfrac{\sqrt{2}}{\pi} lt^{-1}$; $(v_0) = (v_{\iota}) \times (lt^{-1}) = v_0 \div \pi$. Therefore *the maxi-*

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* "Treatise on Electricity and Magnetism," ii, 127.

† Proc. Amer. Philos. Soc., ix, 356, 367, 427, 487, &c.

‡ Proc. Amer. Assoc. Adv. Sci., xxiii, 99.

mum velocity of possible cohesion in our system, is to the minimum velocity of complete dissociation (or the velocity of light), as the diameter of a circle is to its circumference. This relationship points to a kind of circular polarization, induced by the resistance of centres of inertia, as the mediate cause of aggregation by the primitive undulations.

In the following table the principal harmonies which I have pointed out are synoptically shown. It should be remembered that $(v) = (lt^{-1})$ $\propto l^{\frac{1}{2}} \propto t^{-\frac{1}{3}}$; $n = \sqrt{2} \div \pi$; the variants in the right-hand column are symbols of electric dimensions. I have adopted Maxwell's notation with the addition of an accent to mark the symbols of the electro-magnetic system :

$$M = \pi\, n^{-4}\, (M) \propto (v)^4 \propto \mathbb{C}$$
$$a = \pi\, n^{-3}\, (a) \propto (v)^3 \propto \mathbb{C}'$$
$$2r = \pi\, n^{-2}\, (2r) \propto (v)^2 \propto \mathbb{C}',\ \mathfrak{A}$$
$$\beta = \pi\, n^{-1}\, (\beta) \propto (v)^1 \propto \mathbb{C},\ \mathfrak{A}',\ \mathfrak{B}',\ R',\ \mathfrak{D}$$
$$v_0 = \pi\, n^0\, (v_0) \propto (v)^0 \propto E',\ C,\ \mathfrak{B},\ K,\ \mathfrak{D}',\ \mu',\Omega$$
$$v_{\prime} = \pi\, n^1\, (v_{\prime}) \propto (v)^{-1} \propto E,\ C',\ \mathfrak{A}',\ R,\ r'\qquad \Omega'$$
$$v_{\prime\prime} = \pi\, n^2\, (v_{\prime\prime}) \propto (v)^{-2} \propto\qquad L',\ \mathfrak{A},\ K',\ q,\quad \mu$$
$$\gamma = \pi\, n^3\, (\gamma) \propto (v)^{-3} \propto m',\qquad e,\quad p',\ r$$
$$v_{\prime\prime\prime} = \pi\, n^4\, (v_{\prime\prime\prime}) \propto (v)^{-4} \propto m,\ L,\ e',\ p,\quad q'$$

In this table, M = the primary modulus = twice the virtual fall, or the height of a homogeneous æthereal atmosphere at Sun's surface, which would propagate undulations with the velocity of light, the time of virtual fall being half a solar rotation.

$2r$ = Sun's diameter.

v_0 = velocity of light.

$v_{\prime}$ = maximum velocity communicable by solar attraction = $\sqrt{2gr}$ at Sun's surface.

$(v_{\prime})$ = maximum velocity of orbital revolution in our system = $\sqrt{gr}$ at Sun's surface.

$v_{\prime\prime}$ = mean equatorial velocity of radial oscillation with reference to the Central Sun, producing solar rotation.

$(v_{\prime\prime})$ = equatorial velocity of solar rotation.

$v_{\prime\prime\prime}$ = falling velocity communicated, at Sun's equatorial surface, by virtual fall through the half-radius of a circumference equivalent to a red wave-length.

If all the internal resistances of the Sun were converted into motion, the values of (v) and of all its powers would become unity, and *all of the above tabular values would become equivalent to the velocity of light.*

In one of my early papers on the correlations of gravity and magnetism,[*] while seeking experimental evidence of their mutual interdependence, I called attention to the fact that only about $\frac{1}{290}$ of the potential energy of gravity can be converted into actual energy, the re-

* Proc. A. P. S., ix, 356-7.

mainder being opposed by the reaction of molecular elasticity. Maxwell[*] has suggested a crucial experiment of a similar character to the one I then sought. The velocity of his electrified disc bears nearly the same ratio to Earth's orbital velocity, as the diminution of terrestrial attraction by equatorial centrifugal force (or actual energy of superficial gravity) bears to the total attraction. The magnetic disturbance of the disc : Earth's horizontal magnetic force : : the molecular *vis viva* [†] of equatorial rotation : the molecular *vis viva* of orbital revolution.

The molecular oscillation, in alternate approach to and recess from the orbital centre, continues for a half-rotation or a half-revolution, while the terrestrial antagonism lasts only $\dfrac{1}{\pi}$ as long. If we distinguish the terrestrial from the solar units by subscript accents, $l_{\prime} = l$; $t_{\prime} = \dfrac{t}{\pi}$; and, if magnetism and gravitation are tidally related, Maxwell's data[‡] may be represented by the following proportionate tensions :

$m_{\prime}^2\, l_{\prime}^2\, t_{\prime}^{-4} : m^2 l^2 t^{-4} : : m_{\prime}^2 \pi^4 : m^2 : : .128 : 140 \times 144 \times 7000 : : 1 : 1102500000$. Then $m_{\prime}^2 : m^2 : : 1 : 1102500000\pi^4$, and $m = 327710\, m_{\prime}$. This gives a solar parallax of $\sqrt[3]{\dfrac{10^9}{4.432m}} = 8''.83$, which is $\frac{1}{3}$ of one per cent. less than Cornu's parallax.

PLANETARY ILLUSTRATIONS OF THE CREATIVE FIAT.

By Pliny Earle Chase.

(Read before the American Philosophical Society, Aug. 20, 1875.)

In various communications to the American Philosophical Society and to the American Association, I have shown that —

1. The same principles of inertia which cause the Foucault pendulum to record the Earth's rotation, also register the Sun's influence, in sound waves, barometric waves, magnetic variations, mean temperatures, nascent velocities both chemical and cosmical, solar and planetary masses and moments, and stellar and planetary harmonies of relative position, rotation, and revolution.

2. Various independent inertia-estimates of solar distance may be thus obtained, differing from Cornu's final estimate in amounts varying between $\frac{8}{40}$ of one per cent. and $\frac{7}{10}$ of one per cent.

3. All the physical activities which I have tested, seem explicable by ætherial waves, propagated with the velocity of light.

4. Between *a Centauri* and the Sun a parabola can be traced, governed by the solar modulus of light, and determining planetary positions..

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[*] Op. Cit., ii, 370.

[†] The influence of molecular *vis viva* was shown in my discussion of barometric tides, (*ante*, ix, 287). Imray also recognizes its importance in elevating the centre of the molecule, in wave movement, above the normal level, (Proc. Roy. Soc., No. 153, pp. 352–3).

[‡] Ib., ii, 258.